

# Optimization-based tuning of LPV fault detection filters for civil transport aircraft

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## Abstract

In this paper we propose a two steps optimal synthesis approach of robust fault detection filters for the model based diagnosis of sensor faults for an augmented civil aircraft. In the first step a direct analytic synthesis of a linear parameter varying (LPV) fault detection filter is performed for the open-loop aircraft using an extension of the nullspace based synthesis method to LPV systems. The employed scheduling parameters are either directly measurable flight conditions (altitude and speed) or aircraft parameters estimated in real time (mass and center of gravity position). In the second step, a multi-objective optimization problem is solved for the optimal tuning of the LPV detector parameters to ensure satisfactory fault detection performance for the augmented nonlinear closed-loop aircraft. Worst-case global search has been employed to assess the robustness of the fault detection system in the presence of aerodynamics uncertainties and estimation errors in the aircraft parameters. The main appeal of the proposed approach is its applicability to any robustly stable closed-loop aircraft simulation model, for which the open-loop aircraft model is trimmable and linearizable. An application of the proposed method is presented for the detection of failures in the angle-of-attack sensor.

## 1. Introduction

Typical challenging problems in today's advanced flight control systems (FCS) designs for modern aircraft are aiming to significantly reduce the workload of pilots, ensuring best handling qualities simultaneously with increased passenger comfort. Despite a high system complexity, the fault tolerant operation of an aircraft has to be guaranteed over the whole flight envelope in presence of many possible unexpected events and inherent uncertainties in the operation environment. To increase flight autonomy, the aircraft industry traditionally uses (physical) actuator and sensor redundancy. However, this hardware-redundancy based *fault detection and diagnosis* (FDD) approach is becoming increasingly problematic when used in conjunction with the many innovative technical solutions being developed by the aeronautical sector to satisfy the *greener* imperatives demanded by the society.

In the recent years, to alleviate this fault diagnosis bottleneck, efforts have been invested to develop FDD systems which strongly rely on advanced model based FDD techniques. A typical aircraft fault monitoring architecture including a FDD system is depicted in Fig. 1. The augmented closed-loop aircraft structure includes the open-loop aircraft, actuators, sensors and a robust stabilizing controller. A typical FDD system usually includes a residual generator (or fault detection filter, or simply fault detector) to generate the residual signals  $r$ , a residual evaluator to compute  $\theta$  representing an approximation of the residual norm  $\|r\|$ , and a threshold-based decision making block to generate the decision signal  $i$  [2] (e.g., fault if  $i \neq 0$ ). The detector uses as inputs the measurement signals  $y$  and the control input signals  $u$ , which are derived from the measurement vector  $\tilde{y}$  and controller output vector  $\tilde{u}$ , respectively. Depending on the goal of the FDD system, a scalar output residual generator can be used for *fault detection* (FD), whereas a bank of scalar residual generators is required for *fault detection and isolation* (FDI) purposes.

The FDD system must fulfill strong performance specifications which can be expressed in terms of several performance criteria. Therefore, a multi-objective optimization approach appears to be best suited for tuning the free parameters of the FDD system. Typical performance specifications are the *detection time performance* (DTP), *missed detection rate* (MDR), and *false alarm rate* (FAR). However, in the presence of unknown external signals (e.g., pilot inputs, wind disturbances) and parametric uncertainties, these abstract performance criteria are computationally not

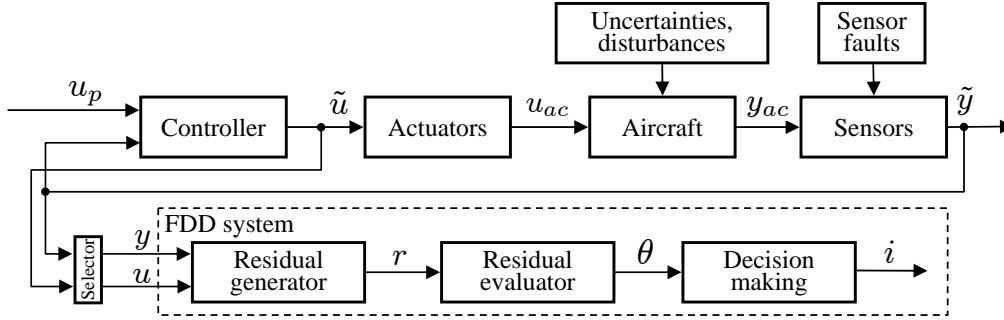


Figure 1: Augmented aircraft model architecture with FDD system

tractable. Therefore, the main challenge of an optimization based design of FDD systems is to turn these performance specifications into computable optimization criteria to support the optimization-based tuning of residual generators and FDD system parameters.

In this paper we propose a two steps optimal synthesis approach of robust fault detection filters for the model based diagnosis of sensor faults for an augmented civil aircraft as depicted in Fig. 1. In the first step, a direct analytic synthesis of a low order *linear parameter varying* (LPV) fault detection filter is performed for the open-loop aircraft using an extension of the nullspace based synthesis method to LPV systems [7]. The employed scheduling parameters are either directly measurable flight conditions (altitude and speed) or aircraft parameters estimated in real time (mass and center of gravity position). This preliminary design allows to define the free parameters of the LPV residual generator, which in the second step are subject to further tuning. In the second step, a multi-objective optimization problem is solved for the optimal tuning of the LPV detector parameters to ensure satisfactory fault detection performance for the augmented nonlinear closed-loop aircraft. Worst-case global search has been employed to assess the robustness of the fault detection system in the presence of aerodynamics uncertainties and estimation errors in the aircraft parameters. The main appeal of the proposed approach is its applicability to any robustly stable closed-loop nonlinear aircraft simulation model, for which the only requirement is that the open-loop aircraft model is trimmable and linearizable.

An application of the proposed method is presented for the detection of failures in the angle-of-attack sensor (shortly  $\alpha$ -sensor) with the help of a low order LPV-gain scheduling based detector. A first order LPV-detector is determined by using recently proposed least order LPV-synthesis methods [7]. Due to the high complexity of the nonlinear aircraft model including highly nonlinear effects which can be encountered during certain maneuvers, in the second step a global, multi objective parameter optimization setup is used for the optimal tuning of the residual generator parameters. The results of the LPV-detector design of the first step serve as initialization of the optimal parameter tuning. Since the criteria evaluation involves the search over a wide parameter space, parallel computation based optimization methods are applied in conjunction with a multi-model approach. The designed LPV residual generator is analyzed in the nonlinear aircraft model of a civil aircraft regarding DTP, FAR and MDR. Worst-case optimizations are used in a parallel computation environment to check the robustness of the design in the defined flight range, taking also into account uncertainties in the aerodynamic database of the aircraft and in the sensor measurements.

## 2. LPV-residual generator synthesis

In this section we formulate the robust sensor fault detection problem for a nonlinear aircraft model and describe a two steps synthesis procedure of LPV residual generators, where the first step is a preliminary structural LPV synthesis using the open-loop aircraft model, while the second step consists of an optimization-based parameter tuning employing the closed-loop robustly stable nonlinear aircraft model. The main advantage of the proposed procedure is that it is applicable to typical aircraft simulation models, where the only requirement is the trimability and linearizability of the open-loop model.

## 2.1 The robust fault detection problem

For the monitored system in Fig. 1, consisting of the open-loop aircraft, actuators and sensors, we assume a nonlinear state-space model description of the form

$$\begin{aligned}\dot{\tilde{x}}(t) &= F(\tilde{x}, \tilde{u}, d, \pi) \\ \tilde{y}(t) &= G(\tilde{x}, \tilde{u}, d, \pi) + D_f f\end{aligned}\quad (1)$$

where  $\tilde{x}$  is the state vector,  $\tilde{u}$  is the control input vector,  $\tilde{y}$  denotes the measured output,  $d$  is an unknown disturbance vector,  $f$  is a sensor fault signal, and  $\pi$  is a constant parameter vector. The vector functions  $F$  and  $G$  are assumed to be differentiable with respect to all intervening variables, thus guaranteeing the existence of solution of (1) as well as of the various Jacobian matrices. The matrix  $D_f$  is simply a column of the identity matrix corresponding to a selected faulty measurement.

Let  $(\tilde{x}_0, \tilde{u}_0, \tilde{y}_0)$  be a plant equilibrium point corresponding to a fixed value of  $\pi$  such that

$$\begin{aligned}0 &= F(\tilde{x}_0, \tilde{u}_0, 0, \pi) \\ \tilde{y}_0 &= G(\tilde{x}_0, \tilde{u}_0, 0, \pi)\end{aligned}\quad (2)$$

In a linearization based gain scheduling approach, the first step is to obtain a linear approximation of the plant around an equilibrium point, where the (measurable) *scheduling variables*, denoted by  $\rho_2$ , explicitly appear. In general,  $\rho_2$  may include components of the measured output  $\tilde{y}_0$  or quantities depending on  $\tilde{y}_0$ , as well as measurable parameters from  $\pi$ . We denote by  $\rho_1$  the non-measurable components of  $\pi$ , and we denote by  $\rho$  the vector with stacked components  $\rho_1$  and  $\rho_2$ . To make explicit the dependence of the equilibrium point on  $\rho$ , we will denote it by  $(\tilde{x}_0(\rho), \tilde{u}_0(\rho), \tilde{y}_0(\rho))$ .

For the aircraft model (1), we can choose  $\rho_2$  with the components including the estimations of the aircraft mass  $m$  and its position of center of gravity  $x_{cg}$ , as well as the measurable outputs flight altitude  $h$  and calibrated airspeed  $V_c$ . The components of  $\rho_1$  usually include the uncertainties in the aerodynamic coefficients as well as the estimation errors in the values of  $m$  and  $x_{cg}$ .

Corresponding to the above equilibrium point, there is an LPV plant model of the form

$$\begin{aligned}\dot{x}(t) &= A(\rho)x(t) + B_u(\rho)u(t) + B_d(\rho)d(t) \\ y(t) &= C(\rho)x(t) + D_u(\rho)u(t) + D_d(\rho)d(t) + D_f f(t)\end{aligned}\quad (3)$$

which describes the local behavior of the nonlinear plant around the equilibrium. In (3),  $x = \tilde{x} - \tilde{x}_0(\rho)$ ,  $u = \tilde{u} - \tilde{u}_0(\rho)$ , and  $y = \tilde{y} - \tilde{y}_0(\rho)$ , and, for example,

$$\begin{aligned}A(\rho) &= \frac{\partial F}{\partial \tilde{x}}(\tilde{x}_0(\rho), \tilde{u}_0(\rho), 0, \pi), \\ B_u(\rho) &= \frac{\partial F}{\partial \tilde{u}}(\tilde{x}_0(\rho), \tilde{u}_0(\rho), 0, \pi), \\ D_d(\rho) &= \frac{\partial G}{\partial d}(\tilde{x}_0(\rho), \tilde{u}_0(\rho), 0, \pi).\end{aligned}$$

In what follows, we assume that  $x(t)$  is the  $n$ -dimensional system state vector,  $y(t)$  is the  $p$ -dimensional system output vector,  $u(t)$  is the  $m_u$ -dimensional control input vector,  $d(t)$  is the  $m_d$ -dimensional disturbance vector, and  $f(t)$  is a scalar fault signal.

The vector  $\rho$  is assumed to belong to a bounded region  $\Pi \subset \mathcal{P}$  of the  $n_\rho$ -dimensional parameter space  $\mathcal{P}$ . In a more general setting we can also allow time-varying parameters  $\rho(t)$ . However, to simplify the notations, the dependence on time of  $\rho$  will not be explicitly emphasized and we consider only slowly varying parameters which can be assimilated with constant values over sufficiently large time periods.

To generate the residual signal  $r(t)$  serving for the decision making on the presence or absence of a fault, a residual generator, processing the measurable output  $y$  and control input  $u$ , has to be designed. For the envisaged synthesis, we employ a parameter dependent gain scheduling filter of the form

$$\mathbf{r}(s) = Q(s, \rho_2) \begin{bmatrix} \mathbf{y}(s) \\ \mathbf{u}(s) \end{bmatrix}\quad (4)$$

where  $\mathbf{r}(s)$ ,  $\mathbf{y}(s)$ , and  $\mathbf{u}(s)$ , are the Laplace transformed vectors  $r(t)$ ,  $y(t)$ , and  $u(t)$ , respectively, with  $r(t)$  a scalar residual signal.  $Q(s, \rho_2)$  is the transfer-function matrix of the filter, depending on the scheduling parameter vector  $\rho_2$ . For a

physically realizable filter,  $Q(s, \rho_2)$  must be *proper* with respect to the Laplace variable  $s$  and robustly *stable* for all values of  $\rho_2$ .

The *Robust Fault Detection Problem* (RFDP) for the detection of a single sensor fault can be formulated as: Determine a proper and stable residual generator having the form (4) such that for all  $\rho \in \Pi$  and given  $\gamma > 0$ , there exists  $\beta > 0$  such that:

$$\begin{aligned} (i) \quad & \|r(t)\| \leq \gamma \max\{\|u(t)\|, \|d(t)\|\} \text{ when } f = 0 \\ (ii) \quad & \|r(t)\| > \beta\|f\| \text{ for } u = 0, d = 0 \\ (iii) \quad & r(t) \text{ is asymptotically bounded.} \end{aligned} \tag{5}$$

where  $\|\cdot\|$  is a suitable signal norm (usually the 2-norm). The requirement (i) expresses the fact that the residual should be as small as possible in the absence of fault. If  $\gamma = 0$  can be used, then the residual  $r$  is exactly decoupled from inputs and disturbances, and therefore condition (i) is also called (approximate) *decoupling condition*. The requirement (ii), called the *detectability condition*, ensures that the fault produces a non-zero residual. The gap defined as  $\beta/\gamma$  measures the sensitivity of the detection task, where larger values guarantee the detection of smaller faults. The condition (iii) is automatically fulfilled provided the underlying closed-loop system is robustly stable. This feature will be assumed in what follows.

## 2.2 Preliminary synthesis of an LPV-filter

The first step of the proposed FDD synthesis approach is the preliminary synthesis of an LPV-filter. For this purpose, we will fix  $\rho_1$  to some representative nominal value and thus consider only the case when  $\rho = \rho_2$ . For synthesis, we employ the recently proposed multi-model based approach of [7] to determine an LPV residual filter of the form (4) which solves the RFDP. For this purpose, we use a family of  $N$  linearized models corresponding to a discrete set of points  $\Pi_N = \{\rho^{(1)}, \rho^{(2)}, \dots, \rho^{(N)}\}$  (obtained, for example, by parameter gridding), to determine a family of  $N$  detectors  $Q(s, \rho^{(i)})$ , for  $i = 1, \dots, N$ . The detectors can be obtained of the form

$$\begin{aligned} \dot{x}_Q^i(t) &= A_Q x_Q^i(t) + B_Q(\rho^{(i)}) \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \\ r^i(t) &= C_Q x_Q^i(t) + D_Q(\rho^{(i)}) \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \end{aligned} \tag{6}$$

and thus, all  $N$  detectors share the same  $A_Q$  and  $C_Q$  matrices. This feature can be easily enforced for scalar output detectors by using a specialized version of the nullspace synthesis method of [8]. Having the grid values  $B_Q(\rho^{(i)})$  and  $D_Q(\rho^{(i)})$ , for  $i = 1, \dots, N$ , interpolation techniques can be employed to determine parametric approximations (e.g., affine or polynomial) of matrices  $B_Q(\rho)$  and  $D_Q(\rho)$ . The above parametric approximations can be determined along the lines of the techniques developed in [4].

The resulting residual generation filter has the following LPV gain scheduling form

$$\begin{aligned} \dot{x}_Q(t) &= A_Q x_Q(t) + B_Q(\rho) \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \\ r(t) &= C_Q x_Q(t) + D_Q(\rho) \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \end{aligned} \tag{7}$$

where the matrices  $A_Q$  and  $C_Q$  are constant matrices and only the matrices  $B_Q(\rho)$  and  $D_Q(\rho)$  depend on the parameter vector  $\rho$ .

The importance of the preliminary synthesis is twofold. Firstly, it serves as a good initial approximation of the final detector allowing the use of more efficient local search based optimization techniques (instead of expensive global search methods, necessary when arbitrary, e.g., random initializations, are used). Secondly, the preliminary synthesis provides a better insight into the structure of the solution of the RFDP (e.g., the zero-nonzero structures in matrices  $B_Q(\rho)$  and  $D_Q(\rho)$ ). This information is very helpful in reducing the number of parameters employed for detector tuning in the next synthesis step.

### 2.3 Multi-objective tuning of the LPV-filter

For the tuning of LPV-fault detection filters in the second step, several performance criteria as *detection time performance* (DTP), *missed detection rate* (MDR) or *false alarm rate* (FAR) must be simultaneously optimized. Depending on the problem to be solved, these criteria could be minimized one at a time and the rest have to remain under given bounds, or a multi-objective optimization problem can be solved. However, due to the presence of unknown external signals (e.g., pilot inputs, wind disturbances) and parametric uncertainties these performance criteria are hardly computationally tractable, the underlying optimization problems being equivalent to optimization problems with semi-infinite constraints.

Due to the high complexity of the nonlinear aircraft model including highly nonlinear effects which can be encountered during certain maneuvers, in the second step a multi-objective parameter optimization setup is used for the optimal tuning of the residual generator parameters. The results of the LPV-detector design of the first step serve as initialization of the optimal parameter tuning. Since the criteria evaluation involves the search over a continuous parameter space, a multi-model approach has been used which strongly favors parallel computations based expensive criteria evaluations, as the simulation based computation of FAR and MDR criteria. The DTP criterion can be included as an explicit constraint in evaluating the MDR.

To define computable approximations of the FAR and MDR criteria, simplifying assumptions are necessary. For example, the minimization of FAR can be seen to be equivalent to determine a detector which ensures the smallest  $\gamma$  to fulfill condition (i) of the RFDP. Instead of arbitrary signals  $u$  and  $d$ , in practical applications it is sufficient to use bounded input signals in a given finite class  $\mathcal{U}$  and bounded disturbance signals in a finite class  $\mathcal{D}$ . A *false alarm bound* can be defined as

$$J_{th}^f = \sup_{\substack{\rho \in \Pi \\ u \in \mathcal{U}, d \in \mathcal{D} \\ f = 0}} \max_{t \leq t_{fin}} \theta(t) \quad (8)$$

The predefined classes  $\mathcal{U}$  and  $\mathcal{D}$  are typical aircraft maneuvers and disturbance signals commonly used to validate the flight controller in nonlinear simulations. The final time  $t_{fin}$  is chosen in accordance with the duration of maneuvers. To avoid false alarms, the threshold  $J_{th}$  used in the decision block must be chosen such that  $J_{th} > J_{th}^f$ .

The minimization of MDR addresses the condition (ii) of the RFDP and can be seen equivalent to maximize the so-called *detection bound* defined as

$$J_{th}^d = \inf_{\substack{\rho \in \Pi \\ u = 0, d = 0 \\ f \in \mathcal{F}}} \max_{t \leq t_{detec}} \theta(t) \quad (9)$$

Here,  $\mathcal{F}$  is a finite set of bounded fault signals of given magnitudes which are selected to be detected within a specified detection time  $t_{detec}$ . Note that the detection times for different fault signals in  $\mathcal{F}$  may even be different. To avoid missed detections, the threshold  $J_{th}$  used in the decision block must be chosen such that  $J_{th} < J_{th}^d$ . To avoid both false alarms and missed detections, the condition  $J_{th}^f < J_{th}^d$  must be guaranteed via an appropriate synthesis of the FDD system.

We can solve this synthesis problem as a simultaneous minimization of  $J_{th}^f$  and maximization of  $J_{th}^d$  over all free parameters of the FDD system. These parameters can be the free parameters of the realization (7) of the LPV detector  $Q(s, \rho)$  as well as parameters  $\eta$  of the evaluation block in Fig. 1 (e.g., weighting terms or filter parameters, see [3]). Thus, the following multi-objective optimization problem has to be solved

$$\min_{Q(s, \rho), \eta} [\epsilon_1 J_{th}^f, -\epsilon_2 J_{th}^d]$$

where  $\epsilon_1$  and  $\epsilon_2$  are suitable scaling weights.

The evaluation of bounds (8) and (9) involves the solution of global optimization problems with respect to the parameter  $\rho \in \Pi$  and therefore, is computationally intensive. To speed up the function evaluations, it is often possible when solving synthesis problems to replace the continuous search domain  $\Pi$  by a finite domain  $\Pi_N = \{\rho^{(1)}, \dots, \rho^{(N)}\}$  containing  $N$  representative points. This replacement also facilitates the use of parallel computations when evaluating the above bounds using simulation-based runs.

## 2.4 LPV-filter robustness assessment and threshold determination

In the above formulation of the synthesis of the fault detection filter, the robustness aspects with respect to the parametric uncertainties in  $\rho_1$  (e.g., in the aerodynamic coefficients, and in the estimated values of scheduling variables) have not been explicitly addressed. Therefore, in general, for an extended parameter space  $\mathcal{P}$  including all uncertainties (i.e.,  $\rho_1$  and  $\rho_2$ ), the corresponding bound  $J_{th}^f$  is higher, while  $J_{th}^d$  is smaller. The decision threshold can be chosen any value  $J_{th}$  satisfying

$$J_{th}^f \leq J_{th} < J_{th}^d \quad (10)$$

Such a choice is always possible by increasing the size of minimum amplitude of faults to be detected. A choice of threshold as  $J_{th} = J_{th}^f + \varepsilon$ , with a small positive  $\varepsilon$ , allows the detection of smaller faults.

For the computation of  $J_{th}^f$  and  $J_{th}^d$ , global worst-case optimization problems have to be solved to find the worst-case parameter combinations. As a robustness measure of the fault detection performance, the smallest *detection gap*  $J_{th}^d - J_{th}^f$  can be used, or alternatively the largest value of the *detection factor*

$$\nu := \frac{J_{th}^f}{J_{th}^d} \quad (11)$$

If  $J_{th}^d - J_{th}^f > 0$  (or equivalently  $\nu < 1$ ), a *constant* threshold can be chosen to serve for fault detection, ensuring no false alarms and no missed detections.

Occasionally a weaker robustness condition can be used for the determination of the decision threshold  $J_{th}$ . A modified detection factor can be defined as

$$\bar{\nu} := \sup_{\rho \in \Pi} \mu(\rho) \leq \nu \quad (12)$$

where  $\mu(\rho)$  is a parameter dependent detection factor defined as

$$\mu(\rho) = \mu_f(\rho) / \mu_d(\rho), \quad (13)$$

with

$$\mu_f(\rho) = \sup_{\substack{u \in \mathcal{U}, d \in \mathcal{D} \\ f = 0}} \max_{t \leq t_{fin}} \theta(t), \quad \mu_d(\rho) = \inf_{\substack{u = 0, d = 0 \\ f \in \mathcal{F}}} \max_{t \leq t_{detec}} \theta(t) \quad (14)$$

If  $\nu > 1$ , but  $\bar{\nu} < 1$ , then we can use a tolerance  $J_{th}(\rho_2)$  depending on the scheduling parameters  $\rho_2$  to solve the RFDP.

## 2.5 Discussion of the proposed approach

The proposed two steps approach for the synthesis of LPV fault detection filters addresses the fault detection problem with minimal modeling requirements. To generate a family of linear models for detector synthesis at Step 1, the only computations needed are the repeated trimming and linearization of the open-loop aircraft model. These features are usually available for any nonlinear aircraft simulation model. For the detector synthesis, ready to use tools are available in the FAULT DETECTION Toolbox [6], which can be used to generate a family of detectors to serve for building LPV fault detection filters. If the underlying domains for the flight conditions and parameter variations are not too large, simpler LPV filters (e.g., with affine dependence of scheduling variables) can be employed. Since the matrices  $A_Q$  and  $C_Q$  of the detector (7) are constant, they can be fixed for the synthesis and any other tuning is performed only on the parameters of the remaining matrices of the LPV-model (7). Note that  $A_Q$  and  $C_Q$  can be easily determined from a single nominal design (using for example the least order nullspace-based synthesis method of [8]) and the resulting detector can be used for the initialization of the tuning process at Step 2. Thus, performing Step 1 can be even skipped. However, this may have also some drawbacks, since useful structural information (e.g., zero-nonzero patterns in the LPV-synthesis) are possibly lost, thus more involved optimizations problems with larger number of free parameters can result.

The multi-objective optimization based synthesis at Step 2 can be performed using standard tools like the MOPS (Multi-objective parameter search) environment of DLR [1]. The *multi-case* feature of MOPS naturally converts any multi-objective optimization problem with  $m$  criteria into a multi-objective optimization with  $mN$  objective functions, where  $N$  is the number of parameter points in  $\Pi_N$  chosen for the synthesis. Since all function evaluations can be performed in parallel, parallel computations can be employed to alleviate the associated computational burden to expensive simulation runs.

### 3. Detection of $\alpha$ -sensor faults

In this section we describe the application of the proposed approach to a nonlinear model of a closed-loop aircraft including a nonlinear control law ensuring robust stability over the whole flight envelope. This model has been previously used in a flight control law clearance study and is described in [5]. As scheduling variables we selected the altitude  $h$ , calibrated speed  $V_c$ , mass  $m$  and position of the center of gravity  $x_{cg}$ , thus  $\rho_2 = (h, V_c, m, x_{cg})$ . The uncertain parameters included in the vector  $\rho_1$  are the aerodynamic coefficients  $C_x, C_y, C_z, C_m, C_n$  and  $C_l$ , and the estimation errors  $\Delta m$  and  $\Delta x_{cg}$  in  $m$  and  $x_{cg}$ , respectively. Thus  $\rho_1$  has 8 components, whose nominal values are zero. For the purpose of illustration of our approach we selected a restricted flight envelope for  $(h, V_c)$  and a restricted domain of variation for  $(m, x_{cg})$ . A grid with  $N = 36$  grid points has been used to generate nominal linearized models. The limits of the chosen envelope and the stepsize between these limits, defining the 36 grid points, are given in Table 1.

Parameter	maximum	minimum	stepsize
Altitude (ft)	15000	25000	5000
Calibrated airspeed (kts)	260	320	60
Mass (t)	180	220	20
Center of gravity (%)	27	33	6

Table 1: Values of the envelope grid

The detection of sensor faults is an important issue for aircraft certification. Typical sensors faults to be detected are low frequency oscillations around the measured values, jamming with the sensor stuck at a constant value, excessive noise (or loss of signal), drift, signal runaway, etc. The exact fault specifications used in this study are given in Table 2. For these faults, a maximum detection time of  $t_{detec} = 1$  sec is required.

Failure type	Parameter
Oscillatory failure	with an amplitude of 5 deg at 0.5Hz
Sensor jamming	at 10 deg
Slow sensor drift	with slope of 4 deg/sec
Fast sensor drift	with slope of 15 deg/sec
Excessive sensor noise	of variance $\sigma^2=80$ deg

Table 2: Set  $\mathcal{F}$  of sensor faults to be detected

In the absence of faults, the robust fault detection filter must work without false alarms in all flight situations. For the estimation of the false alarm bound, a finite set of maneuvers  $\mathcal{U}$  has been selected. Typical maneuvers used are, for example, piloted flights with various pilot inputs (longitudinal/lateral stick doublets, pedal input demand, nose up/nose down demands) or typical navigation maneuvers (level flight, flight path angle target mode, yaw angle target mode, speed change, steady sideslip, coordinated turn, etc.). Typical disturbance inputs used to define  $\mathcal{D}$  are random wind speeds with given variances.

From a fault tolerant control perspective, it is important to detect erroneous behavior of the sensor such that no unintended values are used in the feedback control laws. For a main sensor, as the  $\alpha$ -sensor, triple hardware redundancy is usually provided, which allows to easily detect single sensor faults using a voting based approach. However, the occurrence of two simultaneous  $\alpha$ -sensor faults, although highly improbable, may lead to decreased performance and may require reconfiguration of the control laws. The detection of two simultaneous is more challenging. Our approach to this is to monitor independently each of the three sensors using a model-based fault detection approach and detect the  $\alpha$ -sensor faults using some of the existing measurements and control signals. In this way, all sensor failures can be independently detected and only the fault-free measurements are used in the control laws. The extreme case, when all three sensors fail, can be easily detected and control reconfiguration can be immediately triggered. Alternatively, an estimation of the  $\alpha$ -sensor measurement can be used to replace the true measurements (this possibility is not further explored in this paper).

### 3.1 Preliminary LPV-filter synthesis

The nonlinear open-loop aircraft model has 10 state variables, 22 control inputs, 3 disturbance inputs and 30 measurements. For the synthesis of scalar fault detectors, we used a restricted set of inputs and outputs: 5 control inputs which have direct influence on the angle of attack  $\alpha$ , 3 disturbance inputs, and 6 measured outputs. The rest of variables are assumed to be constant at their trimming values. The inputs vectors  $y$  and  $u$  of the detector are defined as  $y = [\delta\Theta \ \delta\alpha \ \delta q \ \delta\beta \ \delta n_z \ \delta V_{tas}]$  and  $u = [\delta T_l \ \delta T_r \ \delta\eta_l \ \delta\eta_r \ \delta\eta_{stab}]$ , respectively, where the meanings of individual variables are described in Table 3.

Input signals	Description
$\delta\Theta$	pitch angle
$\delta\alpha$	angle of attack
$\delta q$	pitch rate
$\delta\beta$	sideslip angle
$\delta n_z$	vertical load factor
$\delta V_{tas}$	true airspeed
$\delta T_l, \delta T_r$	commanded left and right thrust
$\delta\eta_l, \delta\eta_r$	commanded left and right elevator deflection
$\delta\eta_{stab}$	commanded stabilizer deflection

Table 3: Residual generator input signals

The generated 36 linearized models have been employed to produce 36 first order scalar output detectors of the form (6), which served for the generation of a LPV-detector of the form (7) with constant matrices  $A_Q = -10$  and  $C_Q = 1$ , and the rest of matrices in (7) depending affinely on the scheduling variables  $\rho_2 = (h, V_c, m, x_{cg})$ . Recall that in the preliminary synthesis the nonmeasurable uncertainties were fixed to their nominal values  $\rho_1 = 0$ .

Fig. 2 shows the determination of the detectability factor (13) by simulations for the grid point #3, corresponding to  $m = 180t$ ,  $x_{cg} = 27\%$ ,  $h = 20000\text{ft}$  and  $V_c = 260\text{kts}$ . In the first diagram fault free cases are depicted, where the aircraft is flown through the sets of maneuvers  $\mathcal{U}$  and disturbances  $\mathcal{D}$ . The evaluation signal  $\theta(t)$ , which approximates the 2-norm of the residual signal, remains rather low for all maneuvers and disturbance inputs. In the second diagram  $\theta(t)$  resulting from the slow  $\alpha$ -sensor drift (see Table 2) is depicted.

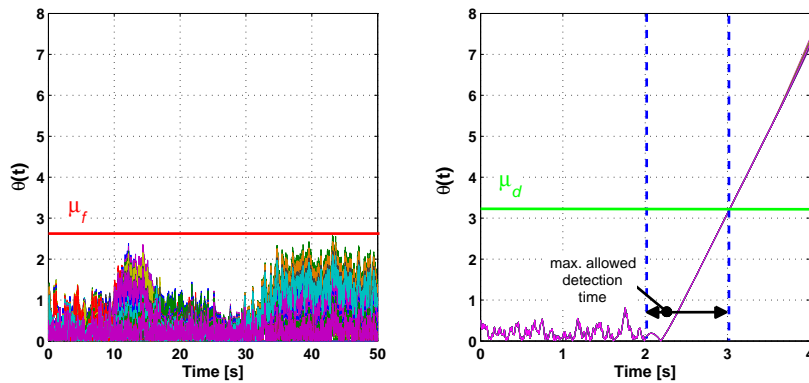


Figure 2: Detectability factor  $\mu < 1$  at grid point #3

As soon as the sensor signal starts drifting away at  $t = 2$  seconds in the simulation,  $\theta(t)$  increases dramatically, allowing the detections the fault in the required time. The slow drift of the sensor signal is chosen as example, as it is usually the most difficult fault to detect due to slow deviation rate. The high amplitudes and high fault rates of the remaining sensor faults described in Table 2 would lead to the stronger robustness condition  $\nu < 1$ , enabling an adequate selection of the threshold  $J_{th}$  and ensuring no false alarms and missed detections. However, for the slow sensor drift, points in the defined range of the envelope can be found where the detectability factor  $\mu > 1$ , avoiding



the suitable selection of  $J_{th}$ . As example where  $\mu_f$  exceeds  $\mu_d$  grid point #36 ( $m = 220t$ ,  $x_{cg} = 33\%$ ,  $h = 25000ft$  and  $V_c = 320kts$ ) is illustrated in Fig. 3.

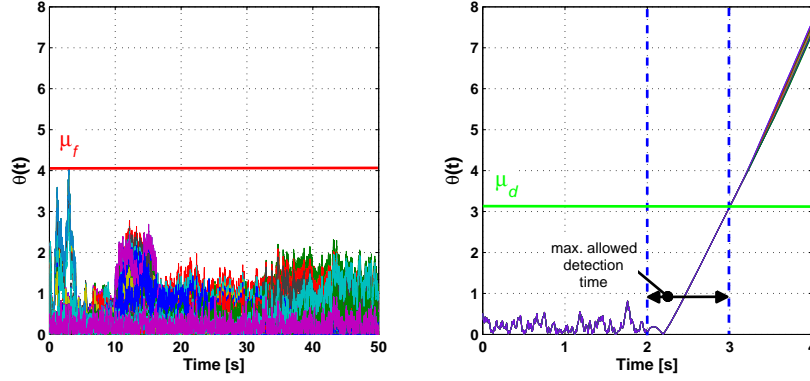


Figure 3: Detectability factor  $\mu > 1$  at grid point #36

To check how severe this phenomena is within the flight range, a worst-case optimization was performed to find the bad values of  $\mu > 1$  and the worst-case  $\bar{\nu}$ . The worst-case search optimization was performed with the global search method *differential evolution* [1]. The results of the worst-case search are presented in Fig. 4. Several regions in the selected flight range are encountered where the detectability factor  $\mu$  exhibits values greater than 1. The criterion values are depicted with different colors, starting from good values ( $\mu \leq 0.5$ ) in blue, passing through cyan ( $0.5 < \mu \leq 0.75$ ), yellow ( $0.75 < \mu \leq 1$ ) to bad values in red ( $\mu > 1$ ). The worst-case of the detectability factor was  $\bar{\nu} = 1.327$  found at  $m = 220t$ ,  $x_{cg} = 33\%$ ,  $h = 25000ft$ ,  $V_c = 317kts$  and  $Ma = 0.73$ . It follows that the FDD system based on the preliminary LPV detector does not fulfill the requirements regarding FAR, MDR and DTP.

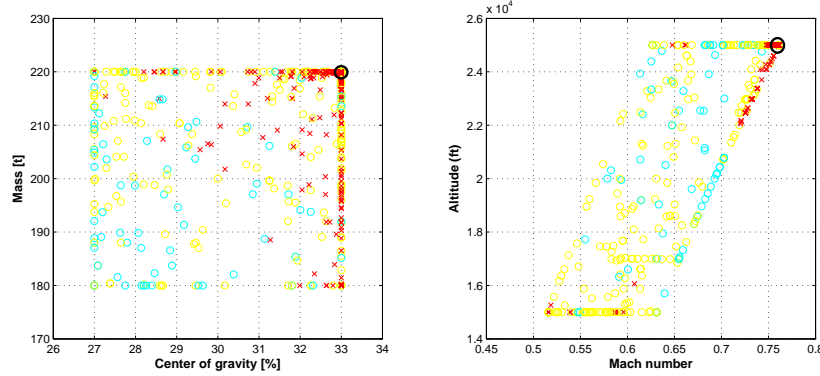


Figure 4: Worst-case search results: points in red violate the robustness requirement  $\mu < 1$

To increase the performance of the residual generator resp. the FDD system and comply with the requirements of FAR, MDR and DTP for the whole set  $\mathcal{F}$  of sensor faults defined in Table 2, a setup for the optimal tuning of the detector parameters is described in the following section. A LPV detector with the same structure as the presented filter in this section is optimized, using the parameter values of the nullspace synthesis as initialization for the tuning.

### 3.2 Optimal tuning of the LPV detector

In this section we describe the approach used for the optimal tuning of the LPV residual filter parameters. The proposed tuning approach is restricted to the region in Table 1, however, it can be easily used to cover the full flight envelope by employing several robust residual generators over similar restricted regions. For the optimal tuning of the parameters of the LPV fault detection filter, derivative free local search optimization software tools have been used. This was possible, taking into account the preliminary synthesis results described previously. The employed optimization tools are available in the MOPS environment of DLR.

The structure of the LPV residual generator as well as the matrices  $A_Q = -10$  and  $C_Q = 1$  are kept as designed during the linear synthesis. The free parameters of the optimization are the polynomial coefficients of the parameter-dependent matrices  $B(\rho)$  and  $D(\rho)$ . For the 11 detector inputs, this amounts to 110 parameters which can be subject to further tuning. To alleviate the computational burden associated with large dimensional search spaces, we exploited the problem structure to reduce the number of parameters subject to tuning. Firstly, all zero elements in the matrices  $B(\rho)$  and  $D(\rho)$  have been frozen to zero values and thus excluded from optimization. Secondly, we selected only the coefficients multiplying the outputs for which the residual is most sensitive in faulty and non-faulty cases. These are the two measurements  $\delta\alpha$  and  $\delta q$  which mainly describe the short period dynamics of the aircraft's longitudinal motion, and  $\delta\beta$  which plays an important role during maneuvers including greater rudder deflections. Additionally, from aircraft symmetry considerations, the inputs  $\delta\eta_l$  and  $\delta\eta_r$  share the same coefficients. In this way, the number of free parameters has been reduced to 21, which allow the use of efficient derivative free optimization methods as the pattern search.

The optimization is a multi-case problem [1], as the continuous search domain  $\Pi$  in (8) resp. (9) is replaced by a finite domain  $\Pi_N = \{\rho^{(1)}, \dots, \rho^{(N)}\}$  containing the  $N = 36$  representative grid points given in Table 1. This replacement also facilitates the use of parallel computations. However, during the a posteriori robustness analysis the continuous search domain  $\Pi$  is considered by using worst-case optimization techniques. An inclusion of such techniques during the optimization would directly guarantee the robustness of the synthesis by fulfilling all criteria, however, would lead to a tremendous increase of the computational burden. The optimization runs are performed applying the options for the local *pattern search* method [1]. The optimization run itself was performed on a Linux-cluster consisting using 16 CPUs. A typical optimization run involving about 1300 function evaluations lasted about 54 hours.

For the assessment of the performance of the resulting FDD system the detectability factor (12) has been determined within the defined flight region. To find the flight envelope parameter values corresponding to  $\bar{\nu}$ , a worst-case optimization as described in section 3.1 was performed. The diagrams in Fig. 5 show the values of the detectability factor  $\mu$  in the flight envelope and the mass and balance diagram, respectively, where the worst-case of  $\bar{\nu} = 0.74$  found at  $m = 220t$ ,  $x_{cg} = 33\%$ ,  $h = 15000ft$ ,  $V_c = 283kts$  and  $Ma = 0.56$  is encircled. Thus, a reliable detection of the fault is possible without false alarms and missed detections.

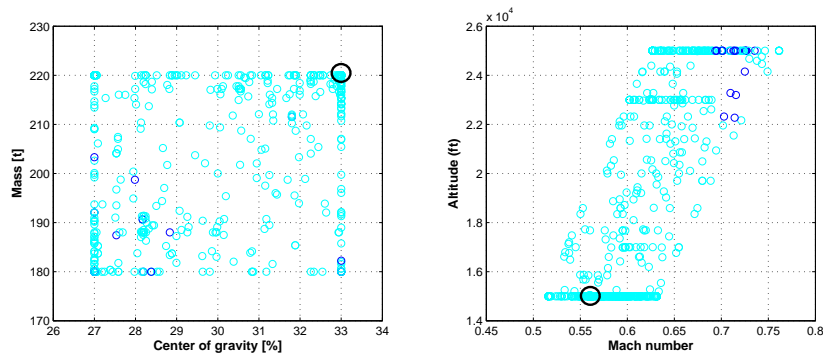


Figure 5: Worst-case search results for the nominal value  $\rho_1 = 0$

To clarify if a constant threshold can be used, the detectability factor (11) has been determined for the optimized residual generator. The result of the worst-case search gives a value of  $\nu = 0.75$ , allowing the selection of a constant threshold in the decision making process. Note that in this case  $\nu \approx \bar{\nu}$ . This is due to the assumption that the fault appears during cruise, where the residual signal is not influenced by any inputs  $u$  and remains low regardless in which flight condition the aircraft is flown. This leads to a similar initial residual signal during the occurrence of a fault.

Finally, uncertainties in the aerodynamic coefficients in the aircraft model up to  $\pm 5\%$  are taken into account, as well as uncertainties of up to  $\pm 10\%$  in the estimations of the mass and position of the center of gravity. Fig. 6 illustrates the worst-case search result of  $\bar{\nu} = 0.79$  for the detectability factor found at  $m = 220\text{t}$ ,  $x_{cg} = 33\%$ ,  $h = 15000\text{ft}$ ,  $V_c = 282\text{kts}$  and  $Ma = 0.56$ .

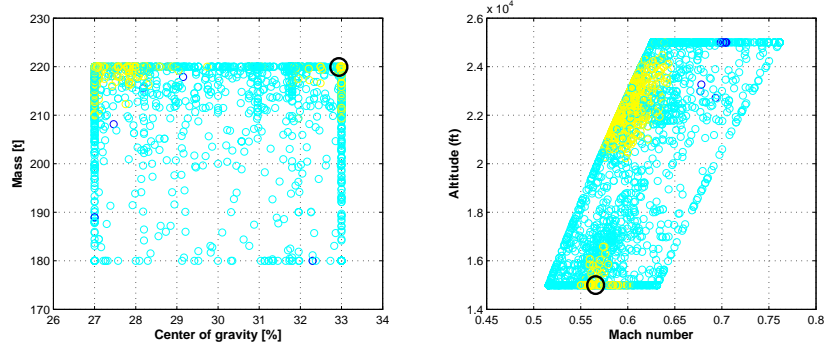


Figure 6: Worst-case search results for  $\bar{\nu}$  including aerodynamic uncertainties and estimation errors

As expected, due the presence of uncertainties the value of  $\bar{\nu}$  is higher than that one for the nominal case. However, no values greater than 1 have been found in the whole envelope for the detectability factor  $\mu$ . Hence the detection of the sensor fault is possible within the required detection time without triggering false alarms. The result of the worst-case search for the robustness measure (11) results in a value of  $\nu = 0.80$ , indicating a robust fault detection system, for which a constant threshold is selectable. For example, the selection of the false alarm bound as threshold  $J_{th} = J_{th}^f$  leads to the worst-case detection times for the individual faults listed in Table 4.

Failure type	Detection Time [s]
Oscillatory failure	0.02
Sensor jamming	0.04
Slow sensor drift	0.88
Fast sensor drift	0.29
Excessive sensor noise	0.02

Table 4: Worst-cases of the detections times for different faults

As mentioned before, the slow sensor drift is the fault which is the most difficult to detect. This is also expressed by the worst-case detections times of the different faults, as the slow sensor drift has the highest detection time. Nevertheless, after the optimization, all defined sensor faults can be detected within the required detection time without triggering false alarms or missed detections, and thus a robust and reliable FDD system can be synthesized for the  $\alpha$ -sensor fault detection problem.

#### 4. Conclusion

The main advantage of the proposed optimal tuning approach of the free parameters of an LPV residual generator for sensor faults is the possibility to perform the tuning using only the provided closed-loop simulation model augmented with the FDD system components. Moreover, for tuning purposes, widely available standard software tools (for trimming, simulation and optimization) are only required. In this way, the use of analytic robust synthesis techniques relying on expensive LPV-modeling of aircraft dynamics can be completely circumvented. This is especially appealing taking into consideration the data confidentiality restrictions of aircraft manufacturers (e.g., regarding controller structure, protection laws, sensor block). This approach could thus provide a way to face the main challenge of any FDD synthesis: the availability of suitable synthesis models.

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